

Effective action for a free scalar field in the presence of spacetime foam

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We model spacetime foam by a gas of virtual wormholes. For a free scalar field we derive the effective Lagrangian which accounts for the interaction with spacetime foam and contains two additional non-local terms. One term describes the scattering of scalar particles on virtual wormholes and explicitly reproduces the Pauli-Villars regularization procedure. The second term describes the back reaction of particles on the number density of wormholes and introduces a self-interaction between particles.

I. INTRODUCTION

At very small scales the spacetime has the foam-like picture [1, 2] which can be modelled by a gas of virtual wormholes [3–5]. We point out that gas of actual (astrophysical) wormholes was first investigated in a series of papers [6–8] where we have demonstrated that actual wormholes may be responsible for the dark matter phenomenon. Unlike astrophysical wormholes virtual wormholes exist only for very small period of time and at very small scales. Such objects (virtual wormholes) were first suggested in Refs. [9–11] where it was proposed that they may lead to loss of quantum coherence. It was latter shown [12–14] that quantum coherence is not lost, since the effects of such wormholes can be absorbed into a redefinition of coupling constants of the low energy theory (see also analogous result in Refs. [15, 16]). However, they still may play an important role in particle physics at very high energies, for they may introduce in a natural way the cutoff at very small scales and may remove divergencies in quantum field theory [3–5, 15]. Moreover, in the presence of external fields the number density of virtual wormholes changes [17, 18] which gives the principle possibility to form wormhole-like objects in laboratory. It also may help to explain a non-vanishing small value of the cosmological constant [3, 4].

In Refs [17, 18] we however considered the case when external fields do not violate the homogeneity of vacuum state which is too restrictive. In the present paper we continue our study by inferring the effective Lagrangian for a free scalar field interacting with the spacetime foam. As we shall see the effective Lagrangian includes two additional non-local terms. One term introduces the linear dispersion (which describes scattering of particles on the foam, i.e., on virtual wormholes). That term is responsible for the removal of divergencies in the theory and it seems to reproduce the well-known Pauli-Willars procedure of the regularization [19, 20]. The second term corresponds to the first nonlinear correction which describes the change of the wormhole number density in an arbitrary (inhomogeneous) external field. Such a term introduces a self-interaction between particles which appears due to the back reaction of the presence of scalar particles on the number density of virtual wormholes. We point out that in the case of a fixed background topology, when the back reaction is absent, the last non-linear term is always absent, i.e. the scalar field remains to be free. However in the case of spacetime foam topology fluctuations change in the presence of scalar particles and such a non-linear term allows to describe the effect of the redistribution of virtual wormholes in external classical fields.

II. VIRTUAL WORMHOLE

In what follows we shall use some of our previous results [3, 4, 16, 18]. A virtual wormhole is described as follows. Consider the metric ($\alpha = 1, 2, 3, 4$)

$$ds^2 = h^2(r) \delta_{\alpha\beta} dx^\alpha dx^\beta, \quad (1)$$

where

$$h(r) = 1 + \theta(a - r) \left(\frac{a^2}{r^2} - 1 \right), \quad (2)$$

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$\theta(x)$ is the step function, and $r^2 = \delta_{\alpha\beta} x^\alpha x^\beta$. Such a wormhole has vanishing throat length. Indeed, in the region $r > a$, $h = 1$ and the metric is flat, while the region $r < a$, with the obvious transformation $y^\alpha = \frac{a^2}{r^2} x^\alpha$, is also flat for $y > a$. Therefore, the regions $r > a$ and $r < a$ represent two Euclidean spaces glued at the surface of a sphere S^3 with the center at the origin $r = 0$ and radius $r = a$. Such a space can be described with the ordinary double-valued flat metric in the region $r_\pm > a$ by

$$ds^2 = \delta_{\alpha\beta} dx_\pm^\alpha dx_\pm^\beta, \quad (3)$$

where the coordinates x_\pm^α describe two different sheets of space. We point out that in the quasi-classical region a virtual wormhole may be taken as a solution of the Euclidean Einstein equations and the function h should be smooth. In particular, the choice $h(r) = (r^2 + a^2)/r^2$ corresponds to the so-called Bronnikov-Ellis metric [21, 22]

$$ds^2 = dR^2 + (R^2 + 4a^2)d\Omega^2,$$

where $d\Omega^2$ is the angular part of the metric, $R = r - a^2/r$ and $-\infty < R < \infty$. There also exist classes of wormhole metrics which possess less symmetry (e.g., for cylindrically and axially symmetric wormholes see [23] and references therein). However as it was shown earlier in [3, 4] for wormholes with the characteristic size of throats $a \ll \ell_{Pl}$ the contribution to the action from the curvature is negligible as compared to the zero-point fluctuations and therefore the model metric (3) is sufficient to our aims. Moreover, in the complete quantum gravity the partition function (which for the scalar field is calculated latter) assumes the sum over all field configurations and, therefore, it is not important which kind of the background metric is used.

Let identify the inner and outer regions of the sphere S^3 and construct a wormhole which connects regions in the same space (instead of two independent spaces). This is achieved by gluing the two spaces in (3) by motions of the Euclidean space (the Poincare motions). If R_\pm is the position of the sphere in coordinates x_\pm^μ , then the gluing is the rule

$$x_+^\mu = R_+^\mu + \Lambda_\nu^\mu (x_-^\nu - R_-^\nu), \quad (4)$$

where $\Lambda_\nu^\mu \in O(4)$, which represents the composition of a translation and a rotation of the Euclidean space. In terms of common coordinates such a wormhole represents the standard flat space in which the two spheres S_\pm^3 (with centers at positions R_\pm) are glued by the rule (4). We point out that the physical region is the outer region of the two spheres. Thus, in general, the wormhole is described by a set of parameters ξ : the throat radius a , positions of throats R_\pm , and rotation matrix $\Lambda_\nu^\mu \in O(4)$.

III. GREEN FUNCTION IN A GAS OF VIRTUAL WORMHOLES

The Green function in a gas of virtual wormholes was considered first in [15]. It was shown there that the exact Green function admits the representation in the form of an infinite series and, therefore, it has too complicated form. In the present paper our primary aim is to investigate the structure and properties of the effective action and, therefore, we will restrict to the simplest case of the minimally coupled scalar field.

Consider now the simplest scalar field and construct the Green function in the presence of a gas of wormholes. The Green function obeys the Laplace equation

$$(-\Delta + m^2) G(x, x') = \delta(x - x')$$

with proper boundary conditions at throats (we require G and the normal derivative $\partial G/\partial n$ to be continuous at throats, where n is the unit vector normal to the surface of the throat section). The Green function for the Euclidean space is merely

$$G_0(x - x') = \frac{m^2}{4\pi^2} \frac{K_1(mr)}{mr} \quad (5)$$

where $r^2 = (x - x')^2$ (and $G_0(k) = 1/(k^2 + m^2)$ for the Fourier transform). In the massless case the Green function reduces to $G_0(x, x') = 1/(4\pi^2 (x - x')^2)$.

In the presence of a single wormhole which connects two Euclidean spaces this equation admits the exact solution. For the outer region of the throat S^3 the source $\delta(x - x')$ generates a set of 4-dimensional multipoles placed in the center of sphere which gives the corrections to the Green function $G_0 + \delta G$. In the present paper we restrict to the lowest monopole term only (see more details in [15, 16]) which gives

$$\delta G_\pm = \mp 2\pi^2 a^2 G_0(r') G_0(r_\pm), \quad (6)$$

where δG_+ corresponds to $r > a$ and $r_+ = r$, while δG_- corresponds to the region $r < a$ and $r_- = a^2/r$. We also assume here $ma \ll 1$ since virtual wormholes are expected to have the Planckian size. A single wormhole which connects two regions in the same space is a couple of conjugated spheres S_\pm^3 of the radius a with a distance $\vec{X} = \vec{R}_+ - \vec{R}_-$ between centers of spheres. So the parameters of the wormhole are $\xi = (a, R_+, R_-)$. The interior of the spheres is removed and surfaces are glued together. Then in the approximation $a/X \ll 1$ the correction to the Green function can be taken as

$$\delta G(x, x') = -2\pi^2 a^2 (G_0(x_+) - G_0(x_-)) (G_0(x'_+) - G_0(x'_-)), \quad (7)$$

where we denote $x_\pm = x - R_\pm$. The above expression explicitly shows the symmetry $x \longleftrightarrow x'$. When we consider a dilute gas approximation the correction to the Green function becomes additive and can be written as

$$\delta G(x, x') = \sum \delta G(x, x', \xi_i) = \int \delta G(x, x', \xi) F(\xi) d\xi, \quad (8)$$

where

$$F(\xi) = \sum_i \delta(\xi - \xi_i) \quad (9)$$

is the density of wormholes in the configuration space ξ . In the vacuum case the background distribution has an isotropic and homogeneous character, i.e., $\rho(\xi) = \langle 0 | F(\xi) | 0 \rangle$ with $\rho(\xi) = \rho(a, X)$, then for the Fourier transforms of the Green function we find

$$G(k) = \frac{1}{k^2 + m^2} \left(1 - \frac{1}{k^2 + m^2} \nu(k) \right) \approx \frac{1}{k^2 + m^2 + \nu(k)}, \quad (10)$$

where

$$\nu(k) = 4\pi^2 \int a^2 (\rho(a, 0) - \rho(a, k)) da, \quad (11)$$

and $\rho(a, X) = \int \rho(a, k) e^{-ikX} \frac{d^4 k}{(2\pi)^4}$.

IV. GENERATING FUNCTIONAL

Consider now the generating functional (the partition function) which is used to generate all possible correlation functions in quantum field theory (and the perturbation scheme when we include interactions)

$$Z_{total}(J) = \sum_\tau \sum_\varphi e^{-S_E} \quad (12)$$

where the sum is taken over field configurations φ and topologies τ (wormholes). The Euclidean action is

$$S_E = \frac{1}{2} (\varphi (-\Delta + m^2) \varphi) - (J\varphi), \quad (13)$$

and we use the notions

$$(J\varphi) = \int J(x) \varphi(x) d^4 x.$$

Here J denotes an external current. The sum over field configurations φ can be replaced by the integral

$$Z^*(J) = \int [D\varphi] e^{\frac{1}{2}(\varphi(\Delta - m^2)\varphi) + (J\varphi)}. \quad (14)$$

Upon the simple transformations

$$\frac{1}{2} (\varphi (-\Delta + m^2) \varphi) - (J\varphi) = \frac{1}{2} (\tilde{\varphi}(-\Delta + m^2)\tilde{\varphi}) - \frac{1}{2} (JGJ), \quad (15)$$

where $\tilde{\varphi} = \varphi - GJ$ and G is the background Green function $G = G_0 + \delta G(\xi)$, e.g., see (5) and (8), we cast the partition function to the form

$$Z^* = \int [D\tilde{\varphi}] e^{-\frac{1}{2}(\tilde{\varphi}(-\Delta+m^2)\tilde{\varphi}) + \frac{1}{2}(JGJ)} = Z_0(G) e^{\frac{1}{2}(JGJ)}, \quad (16)$$

where $Z_0(G) = \int [D\varphi] e^{-\frac{1}{2}(\varphi(-\Delta+m^2)\varphi)}$ is the standard expression and $G = G(\xi_1, \dots, \xi_N)$ is the Green function for a fixed topology, i.e., for a fixed set of wormholes ξ_1, \dots, ξ_N .

Consider now the sum over topologies τ . To this end we restrict with the sum over the number of wormholes and integrals over parameters of wormholes:

$$\sum_{\tau} \rightarrow \sum_N \int \prod_{i=1}^N d\xi_i = \int [DF] \quad (17)$$

where F is given by (9). We point out that in general the integration over parameters is not free (e.g., it obeys the obvious restriction $|\vec{R}_i^+ - \vec{R}_i^-| \geq 2a_i$). This defines the generating function as

$$Z_{total}(J) = \int [DF] Z_0(G) e^{\frac{1}{2}(JGJ)}. \quad (18)$$

Expanding this expression by J we find

$$W(J) - W(0) = \ln \frac{Z_{tot}(J)}{Z_{tot}(0)} \approx \frac{1}{2} \overline{(JGJ)} + \frac{1}{8} \overline{(J\Delta GJ)^2} + \frac{1}{48} \overline{(J\Delta GJ)^3} + \dots \quad (19)$$

where overbar denotes vacuum mean value $\overline{G} = \langle 0|G|0 \rangle$, i.e.,

$$\overline{G} = \langle 0|G|0 \rangle_{J=0} = \frac{1}{Z_{total}(0)} \int [DF] Z_0(G) G(\xi).$$

The two terms in (19) can be expressed via moments of the density of wormholes in the configuration space as follows

$$\overline{G} = G_0 + \int \delta G(\xi) \rho(\xi) d\xi,$$

where

$$\rho(\xi) = \langle 0|F(\xi)|0 \rangle_{J=0}$$

is the mean density (9) and $\delta G(\xi)$ is given by (7). Analogously the next term in (19) which describes topology fluctuations can be expressed as

$$\overline{\Delta G \Delta G} = \int \delta G(\xi) \delta G(\xi) \rho(\xi) d\xi + \int \delta G(\xi) \delta G(\xi') \omega(\xi, \xi') d\xi d\xi'$$

where we denote $(\Delta F(\xi) = F(\xi) - \rho(\xi))$

$$\rho(\xi, \xi') = \langle 0|\Delta F(\xi) \Delta F(\xi')|0 \rangle_{J=0} = \rho(\xi) \delta(\xi - \xi') + \omega(\xi, \xi').$$

All higher order mean values in (19), e.g., $\overline{\Delta G \Delta G \Delta G}$, are expressed via the respective momenta $\rho(\xi, \xi', \xi'')$, etc. as

$$\overline{\Delta G \Delta G \Delta G} = \int \delta G(\xi) \delta G(\xi') \delta G(\xi'') \rho(\xi, \xi', \xi'') d\xi d\xi' d\xi''.$$

We use the approximation of a rarefied gas of virtual wormholes and, therefore, to the leading order we may assume that the correlations between positions of wormholes are absent, i.e., $\omega(\xi, \xi') \approx 0$. To account for such correlations it requires the further development of the theory. Thus, we find the decomposition

$$W(J) - W_0 = \frac{1}{2} (JG_0J) + \frac{1}{2} \int (J\delta G(\xi)J) \rho(\xi) d\xi + \frac{1}{8} \int (J\delta G(\xi)J)^2 \rho(\xi) d\xi + \dots \quad (20)$$

The first term here corresponds to the standard free scalar field. The second term describes effects of the scattering of scalar particles on the non-trivial topology. We point out that if the topology does not change by the presence of scalar particles (i.e., it is rigidly fixed, though it may be random) next terms do not appear at all. While in general there appears a non-linearity (due to the back reaction). In other words, in the presence of spacetime foam the scalar field inevitably acquires a self-interaction (the last term above) which means a modification of the field theory. The more convenient way to analyze such a modification is to consider the effective action.

V. EFFECTIVE ACTION

Consider the mean vacuum value of the scalar field which is given by $\varphi_c(J) = \langle 0|\varphi|0 \rangle_J = \frac{\delta W}{\delta J}$ or

$$\varphi_c(J) = G_0 J + \int \delta G(\xi) J \rho(\xi) d\xi + \frac{1}{2} \int (J \delta G(\xi) J) \delta G(\xi) J \rho(\xi) d\xi + \dots$$

This equation can be resolved $J(\varphi_c)$, then we define the effective action

$$\Gamma(\varphi_c) = (\varphi_c J(\varphi_c)) - W(J(\varphi_c)).$$

This transformation can be carried out by the perturbation method. Indeed, $\delta G(\xi)$ includes a small parameter a^2 which is expected to have the order $a \sim \ell_{Pl}$. Thus we find

$$\Gamma(\varphi) = \frac{1}{2} (\varphi (-\Delta + m^2) \varphi) + V_1(\varphi) + V_2(\varphi) + \dots$$

where

$$V_1(\varphi) = -\frac{1}{2} \int \varphi (-\Delta + m^2) \delta G(\xi) (-\Delta + m^2) \varphi \rho(\xi) d\xi$$

and

$$V_2(\varphi) = -\frac{1}{8} \int (\varphi (-\Delta + m^2) \delta G(\xi) (-\Delta + m^2) \varphi)^2 \rho(\xi) d\xi.$$

In the explicit form (by means of use (7)) we find

$$V_1(\varphi) = \pi^2 \int a^2 (\varphi(R_+) - \varphi(R_-))^2 \rho(\xi) d\xi$$

and

$$V_2(\varphi) = -\frac{\pi^4}{2} \int a^4 (\varphi(R_+) - \varphi(R_-))^4 \rho(\xi) d\xi.$$

We point out that this term is negative which reflects the well-known fact that the second order correction to the ground state is always negative.

Next terms describe higher order corrections, e.g., the multipole contribution in (7) and correlations between positions of wormholes, e.g.,

$$V_3(\varphi) = -\frac{\pi^4}{2} \int a^2 (\varphi_+ - \varphi_-)^2 a'^2 (\varphi'_+ - \varphi'_-)^2 \omega(\xi, \xi') d\xi d\xi'.$$

Now we are ready to analyze the effective potential $U = V_1 + V_2$. The first term V_1 defines the renormalization of the Green function in the form (11) which reflects the scattering of scalar particles on virtual wormholes. Nevertheless for the low energy theory it is convenient to expand this potential in series. Indeed, the definition of U includes the unknown function $\rho(\xi)$ which requires the further studying. In the vacuum state the space remains to be isotropic and homogeneous. Therefore, we may expect that $\rho(\xi)$ has the structure $\rho(\xi) = \rho(a, |X|)$, where $X = R_+ - R_-$, and V_1 can be rewritten as

$$V_1(\varphi) = \pi^2 \int a^2 (\varphi(x+X) - \varphi(x))^2 \rho(a, X) da d^4 X d^4 x.$$

This part explicitly reduces to the correction (11). Nevertheless it has sense to consider its decomposition. For the spacetime foam we expect the typical values $X, a \sim \ell_{Pl}$ and this allows to expand V_1 by the small parameter X as

$$V_1(\varphi) = \pi^2 \int a^2 \left(\varphi(x) \left(1 - e^{-X^\mu \nabla_\mu} + 1 - e^{X^\mu \nabla_\mu} \right) \varphi(x) \right) \rho(a, X) da d^4 X d^4 x,$$

$$V_1(\varphi) = -2\pi^2 \sum_{n=1}^{\infty} \int a^2 \varphi(x) \left(\frac{X^{2n}}{(2n)!} (s^\mu \nabla_\mu)^{2n} \right) \varphi(x) \rho(a, X) da d^4 X d^4 x,$$

where $s^\mu = X^\mu/X$, which gives

$$V_1(\varphi) \approx -\frac{1}{2} \int \varphi \left(A\Delta + \frac{1}{M_1^2} \Delta^2 + \dots \right) \varphi d^4x. \quad (21)$$

Here coefficients are

$$A = \frac{\pi^2}{2} \int a^2 X^2 \rho(a, X) da d^4X = \frac{\pi^2}{2} \langle a^2 X^2 \rangle_n$$

$$\frac{1}{M_1^2} = \frac{\pi^2}{16} \int a^2 X^4 \rho(a, X) da d^4X = \frac{\pi^2}{16} \langle a^2 X^4 \rangle_n$$

where $n = \int \rho(a, X) da d^4X$ is the density of wormholes and we have used properties of the isotropy of the vacuum distribution $\rho(\xi)$, i.e.,

$$\langle s^\alpha s^\beta \rangle = \frac{1}{4} \delta^{\alpha\beta},$$

$$\langle s^\alpha s^\beta s^\mu s^\nu \rangle = \frac{1}{24} (\delta^{\alpha\beta} \delta^{\mu\nu} + \delta^{\alpha\mu} \delta^{\nu\beta} + \delta^{\alpha\nu} \delta^{\beta\mu}).$$

Rigorously speaking the second term in (21) should also include contribution from 4-dimensional dipole component in (7). However the symmetry of the vacuum means that such a term has the same structure with a coefficient $1/M^2 \sim \langle a^4 X^2 \rangle_n$. It is clear that such a contribution can be absorbed in the coefficient M_1 .

In the same way we find the decomposition of the self-interacting term as

$$V_2(\varphi) = -\frac{1}{4!} \int \frac{1}{M_2^4} (\nabla\varphi)^4 d^4x + \dots$$

where

$$\frac{1}{M_2^4} = \frac{\pi^4}{6} \langle a^4 X^4 \rangle_n.$$

Thus for the effective Lagrangian we get

$$\Gamma(\varphi) = \int \left[\frac{1}{2} \varphi \left(-\Delta \left(1 + A + \frac{1}{M_1^2} \Delta \right) + m^2 \right) \varphi - \frac{1}{4! M_2^4} (\nabla\varphi)^4 \right] d^4x + \dots \quad (22)$$

We point out that the specification of all possible momenta (i.e., of all constants in the expression above) is simply an equivalent way to the definition of the vacuum distribution of virtual wormholes $\rho(\xi)$. Besides, since this expansion involves the infinite series of constants, the resulting theory belongs formally to the class of non-renormalizable theories (as quantum gravity is).

VI. DISCUSSIONS

Consider first the quadratic part $\Gamma_0(\varphi)$ of the effective action (22). The dimensionless parameter A merely renormalizes the naked values of m and φ . Therefore, this constant can be excluded, e.g., we may take $A = 0$. The naive estimate for the rest parameters is $M_a \sim M_{Pl} \gg m$. In particular, Abdo et al. [24] have recently found a very stringent bound $l_1 < L_{Pl}$, where L_{Pl} is the Planck length, on the first nontrivial correction to the standard photon dispersion relation $\omega^2 = k^2(1 + kl_1 + k^2 l_2^2 + \dots)$. The isotropy and homogeneity of vacuum forbid corrections of the type kl_1 which means that the first correction is quadratic. Nevertheless, we can accept such an estimate, then the effective action can be recast into the form

$$\Gamma_0(\varphi) \approx \frac{1}{2} \int \left(k^2 + m^2 - \frac{1}{M_1^2} k^4 + \frac{1}{M_3^4} k^6 + \dots \right) |\varphi_k|^2 \frac{d^4k}{(2\pi)^4}. \quad (23)$$

If we neglect all terms higher than k^4 then we get

$$\Gamma_0(\varphi) \approx \frac{1}{2} \int \left((k^2 + \tilde{m}^2) \frac{1}{-M_1^2} (k^2 + \tilde{M}_1^2) \right) |\varphi_k|^2 \frac{d^4 k}{(2\pi)^4} + \dots$$

where $\tilde{M}_1^2 = -M_1^2(\sqrt{1 + 4m^2/M_1^2} + 1)/2 \approx -M_1^2$ and $\tilde{m}^2 = M_1^2(\sqrt{1 + 4m^2/M_1^2} - 1)/2 \approx m^2$. Let us forget for a while that one mass has the negative sign $\tilde{M}_1^2 \approx -M_1^2$. We see that at sufficiently small energies $k^2 \ll M_1^2$ the above action describes the standard free scalar particles and effects of the spacetime foam are negligible (they are merely encoded in the renormalized parameters). While at Planck energies (due to the interaction with the foam) there appear in the theory new (additional) particles with a very huge mass $\tilde{M}_1 \sim M_{Pl}$. From the formal standpoint this part recovers the well-known Feynman or Pauli-Villars regularization procedure [19, 20]. It becomes also clear the general structure of the above action which can also be seen from (11) (e.g., $\nu(k) = \sum (\pm L_n^2 k^2)^n$). Next terms of the decomposition of $V_1(\varphi)$ will add new additional and more massive terms which generate additional degrees of freedom, e.g.,

$$\Gamma_0(\varphi) = \frac{1}{2} \int \varphi (-\Delta + \tilde{m}^2) \prod_a \left(\frac{-\Delta + \tilde{M}_a^2}{\tilde{M}_a^2} \right) \varphi d^4 x \quad (24)$$

where \tilde{m} , \tilde{M}_a are the physical (i.e., already renormalized) values of the mass spectrum for the scalar particles in the theory.

This action contains a number of extra degrees of freedom which are very heavy particles with masses $\tilde{M}_a \gtrsim M_{Pl}$. Such additional particles relate to the modes defined on the wormhole necks and more complex collective excitations. Indeed, if we consider a constant time sections then, in general, such a section splits into the Euclidean 3-dimensional space and a number of closed spaces (throats or baby universes). Since all such throats have the Planckian size then the respective modes have wavelengths which start from Planckian values. This explains why such additional particles are extremely heavy.

The only annoying thing with the above action is that some part of spectrum has imaginary (or imaginary parts) masses, e.g., $\tilde{M}_1^2 \approx -M_1^2 < 0$. For particle physics (upon continuation to the Minkowsky sector) this means the presence of instabilities, the energy of particles $\varepsilon = \sqrt{p^2 - M_1^2}$ is imaginary as $p < M_1$. Such an instability represents the well-known and essential property of wormholes which corresponds to the negative mode found first in Refs. [25, 26]. It describes the instability of a large universe against the emission of baby universes and may serve as one of the simplest examples of theories which admit solutions violating the null energy condition [27] (see also references therein).

The stable vacuum corresponds to the case when all masses have values $\tilde{M}_a^2 > 0$. Frankly speaking we do not know what kind of Planck-scale physics is hidden here and may only suggest some speculations. The first and simplest possibility is that higher order corrections should include multipoles of higher orders in (7) which may properly change the values of the mass spectrum. This however is too optimistic point of view. Indeed, baby universes correspond to the virtual wormholes with sufficiently long necks. Then the monopole component is absent and the first contribution in (7) comes from the dipole. Nevertheless, the instability retains as it was shown in [25, 26]. The second possibility is that the restriction to a finite number of terms in the above action is simply incorrect. Indeed, if we take, as an example, a particular form of $\rho(\xi)$, e.g., see [16, 18]

$$\rho(\xi) = n\delta(a - a_0) \frac{1}{2} (\delta^4(X - r_0) + \delta^4(X + r_0)),$$

we find $\nu(k) = 4\pi^2 n a_0^2 (1 - \cos(kr_0)) \geq 0$. Then in the general case when $n = n(a_0, r_0)$ we get

$$\nu(k) = \int n(a, X) a^2 (1 - \cos(kX)) da dX \geq 0$$

which means that the quadratic part of the action

$$\Gamma_0(\varphi) = \frac{1}{2} \int (k^2 + m^2 + \nu(k)) |\varphi_k|^2 \frac{d^4 k}{(2\pi)^4}$$

actually contains no poles on the real axis k (at least in the Euclidean sector). Though, formally the expansion of $\cos(rk)$ gives the series of the alternating sign (exactly as in (23)). This however does not guarantee the absence of poles on the complex plane k and the stability of the vacuum (the stable vacuum contains only poles of the

type $k_a = \pm i\widetilde{M}_a$). Here we come to the third possibility that the instabilities have the direct physical sense and lead to baby universes production [25, 26]. Such an instability should be accompanied with the phase transitions. Fortunately we have the additional self-interacting terms, e.g., V_2 . The instability results in the particle (as well as actual wormholes or baby universes) production and the change (redefinition) of the vacuum state (which is described by the Bogolubov's transformations for an appropriate particles \widetilde{M}_a). In other words such an instability (and actual wormhole production) works till the moment when all masses get into the stable physical sector $Im\widetilde{M}_a = 0$. Then the quadratic part of the action transforms exactly to the Pauli-Villars type (24).

Consider now the second self-interacting term and continuation to the Minkowsky space $S = i\Gamma_0(\varphi)$, $(\nabla\varphi)^2 \rightarrow -(\partial_\mu\varphi)^2$, $-\Delta \rightarrow (\partial_\mu)^2$, etc. Then in the low energy limit we find (we neglect the higher order corrections in the quadratic part)

$$S(\varphi) = \int \left[\frac{1}{2} \left((\partial_\mu\varphi)^2 - m^2\varphi^2 \right) + \frac{1}{4!M_2^4} \left(-(\partial_\mu\varphi)^2 \right)^2 \right] \sqrt{-g}d^4x$$

where $(\partial_\mu\varphi)^2 = g^{\mu\nu} (\partial_\mu\varphi) (\partial_\nu\varphi)$, etc. The non-linear term here describes the redistribution of virtual wormholes (the change in the wormhole number density) in an external field. We point out that such a term does not presume the homogeneity of the external field as in Refs. [17, 18].

Consider now the presence of an external field φ_0 which should be a solution to the equations of motions which follow from the above action, i.e., $\delta S(\varphi_0) = 0$. Then we find the quadratic part of the action in the presence of the external field φ_0 as

$$S_2(\varphi_0, \varphi) = \frac{1}{2} \int [(\Gamma^{\mu\nu}(\varphi_0)\partial_\mu\varphi\partial_\nu\varphi - m^2\varphi^2)] \sqrt{-g}d^4x$$

where

$$\Gamma^{\mu\nu}(\varphi_0) = \left(1 + \frac{1}{3!M_2^4} n_\alpha n^\alpha \right) g^{\mu\nu} + \frac{2}{3!M_2^4} n^\nu n^\mu$$

and $n_\mu = \partial_\mu\varphi_0$. Let the external field has the property $\partial_\mu\varphi_0 = n_\mu = const$. Then $\Gamma^{\mu\nu}(\varphi_0) = const$ and from the above action we find the dispersion relations in the form (compare to the consideration in Ref. [16])

$$\Gamma^{\mu\nu}(\varphi_0)k_\nu k_\mu - m^2 = 0$$

Consider the time-like vector $n^\mu = (b, 0)$ and $k_\mu = (\varepsilon, \mathbf{p})$. Then we find $n_\alpha n^\alpha = b^2$ and this gives (we assume $\frac{b^2}{3!M_2^4} \ll 1$)

$$\varepsilon^2(\mathbf{p}) \approx \left(1 - \frac{3b^2}{3!M_2^4} \right) m^2 + \left(1 - \frac{2b^2}{3!M_2^4} \right) \mathbf{p}^2.$$

We see that in this case the speed of light does not exceed the vacuum value

$$c^2 = 1 - \frac{2b^2}{3!M_2^4} < 1$$

which, according to [16], means that the additional density of virtual wormholes is $\delta n > 0$.

Consider now the space-like vector $n^\mu = (0, \mathbf{b})$ which gives $n_\alpha n^\alpha = -\mathbf{b}^2$ and therefore the dispersion relations takes the form

$$\varepsilon^2(\mathbf{p}) \approx m^2 \left(1 - \frac{\mathbf{b}^2}{3!M_2^4} \right) + \mathbf{p}_\perp^2 + \left(1 - \frac{3\mathbf{b}^2}{3!M_2^4} \right) \mathbf{p}_\parallel^2$$

which gives $c_\perp^2 = 1$ and $c_\parallel^2 = 1 - \frac{\mathbf{b}^2}{3!M_2^4} < 1$. According to [16] this means $\delta n < 0$. In other words the external field may either increase, or suppress the density of virtual wormholes. There remains a much more complex situation when the external field retains the dependence on coordinates which however requires the further investigations.

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